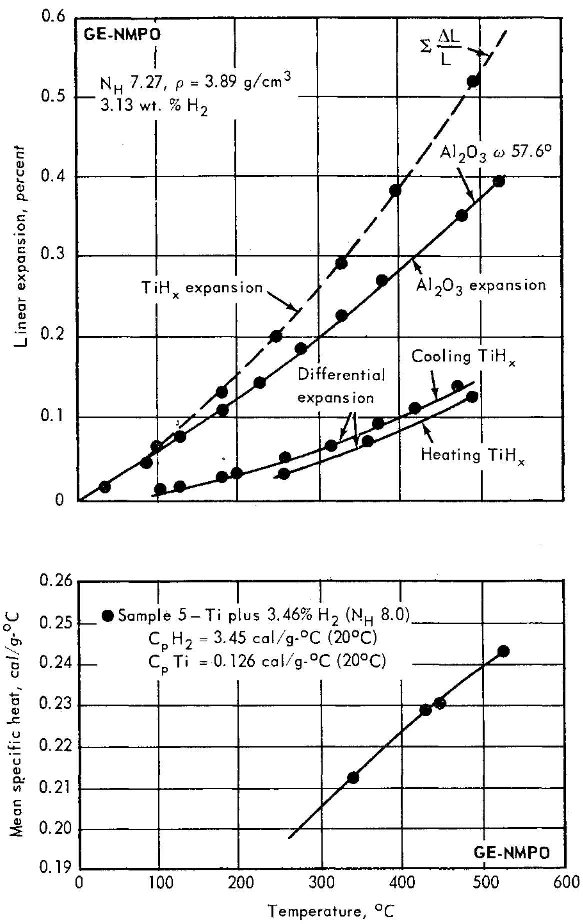


**Fig. 3 Linear thermal expansion and specific heat of hydrided titanium.**

Power (AEC SNAP) 2, 8, and 10A reactor programs. Among the latest publications is one by Beck.[5]

Table 1 shows that the density of this material (5.6 g/cm³) is the highest of the metal hydride materials listed and that it has an 800°C temperature limitation. By appropriate additions and the subsequent metallurgical working of the structure, a fine prehydrided grain size can be produced. The fine grain size combined with the proper hydriding cycle will produce sound, small-grained Zr bodies of $N_H$ 7.0, although $N_H$ 6.0 bodies are more practical for engineering applications.

### *Thermal stability*

Unclad samples of $ZrH_x$ ($N_H$ 6.0) retained their dimensional stability and hydrogen (>95%) after >1000 hr in Mach 0.2 air at 650°C.

Seven samples (~1.5 cm in diameter × 2.0 cm in length–$N_H$ 6.6) were included in the 540°C thermal stability experiment on titanium hydride previously described. The weight gain rates and the average parabolic oxidation rate constant for the 2522-hr test period are approximately three and ten times greater, respectively, than the values for hydrided titanium and are very similar to the values for unalloyed zirconium at this temperature.[2]

## Concluding Remarks

Of the three metal hydrides discussed in this paper, acceptance of zirconium is by far the most advanced relative to nuclear applications. As early as the late 1950's, complete reactor tests were run, employing hydrided zirconium[4] and hydrided zirconium-uranium.[3] Since these tests, SNAP 2, 8, and 10A programs have announced the use of a hydrided

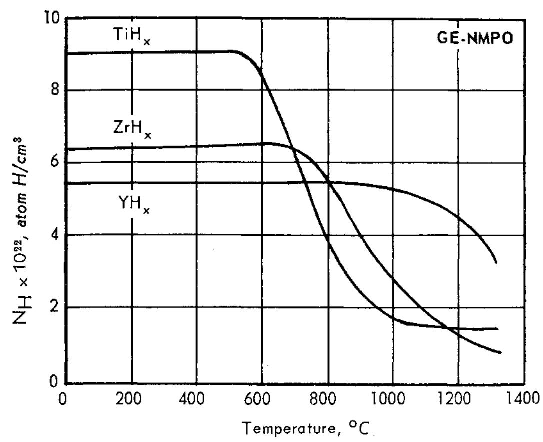


**Fig. 4 Content of absorbed hydrogen in metallic zirconium, titanium, and yttrium in equilibrium with 15 psia of hydrogen at various temperatures.**

Zr-U alloy as a reactor fuel element. At present there are no known reactors in operation or under construction employing yttrium or titanium hydride. However, it is believed that, as more engineering and physical property data on massive metal hydrides are obtained, these materials will be accepted by design engineers for application as nuclear shield and moderator materials in advanced powerplants.[2]

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# Atmospheric Density Variations with Latitude and Season


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VARIATIONS in atmospheric density are of increasing importance in the design of aircraft, missiles, and space boosters. This note presents a diagram and a table, which relate average density and density variation for January and July at latitudes over the Northern Hemisphere to the


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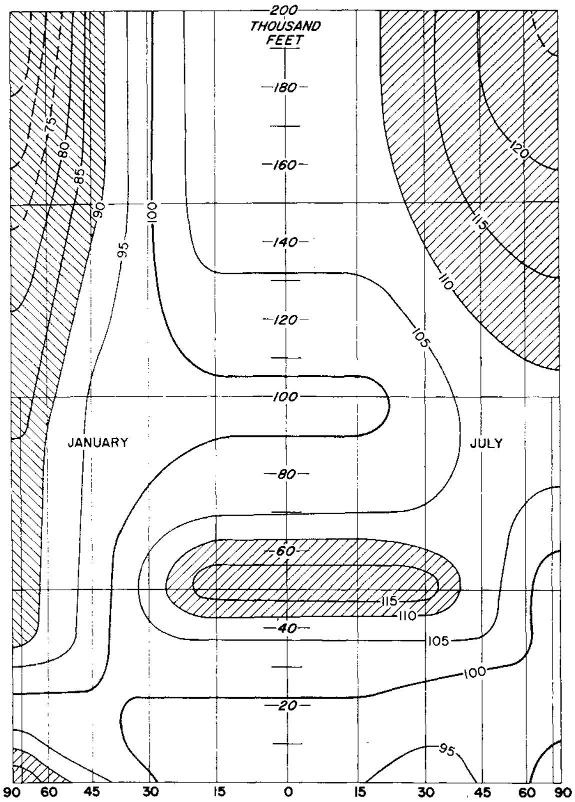


**Fig. 1 Atmospheric density in January and July, as percent of that at the same level in the U. S. Standard Atmosphere.[1] Basic density data derived from a family of supplemental atmospheres[2] for each 15° of latitude.**

densities given in the *U. S. Standard Atmosphere, 1962*,[1] hereafter designated USSA. Latitudes are plotted on a sine scale to make them proportional to the surface area of the earth between latitude parallels. This device permits estimation of the fraction of the earth's atmosphere having a given percentage of standard density.

Figure 1, representing density as percentage of USSA density, was prepared from the data given in Table 1. These data were derived from a family of "supplemental atmospheres" developed as adjuncts to the USSA for its sponsor, the Committee on Extension to the U. S. Standard Atmosphere. The USSA, which depicts middle-latitude conditions averaged over longitude and season, has replaced, for purposes of engineering design and comparison, the 1952 U. S. Standard and 1956 extension thereto, as well as the related Air Research and Development Command (ARDC) model atmospheres of 1956 and 1959. The supplemental atmospheres depict average conditions in January and July, separately, at latitudes 75, 60, 45, and 30 N, and for the year as a whole at 15 N, where seasonal variation is slight[2,3]; the 75 N atmospheres extend to 31 km, and all of the others extend to 90 km (295 kft).

Densities for exact geometric heights, rather than geopotential heights, are given in Table 1 as percentages of USSA values. They can be converted to percentages of the 1959 ARDC values by dividing by the appropriate 1959 ARDC percentage in the last column. The ± increments are estimated 95% range variabilities, as discussed below.

### Variability

To estimate variabilities, three additional sources were used: 1) an Air Force Survey in Geophysics[4] giving standard deviations of density about monthly mean values at 2 km intervals over six stations (years 1958–1960), 2) the Army

**Table 1 Atmospheric densities as percentages of density at corresponding height in *U. S. Standard Atmosphere, 1962***

| Height, $10^3$ ft | USSA, lb/ft³ | January 75N | January 60N | January 45N | January 30N | January 15N | July 15N | July 30N | July 45N | July 60N | July 75N | ARDC, 1959 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 7.6474 −2[a] | 116 ± 8 | 113 ± 7 | 106 ± 6 | 100 ± 3 | 95 ± 2 | 95 ± 2 | 95 ± 2 | 97 ± 3 | 100 ± 4 | 104 ± 4 | 100 |
| 10 | 5.6483 −2 | 106 ± 5 | 103 ± 5 | 102 ± 4 | 98 ± 2 | 95 ± 2 | 95 ± 1 | 95 ± 1 | 98 ± 2 | 99 ± 2 | 100 ± 2 | 100 |
| 20 | 4.0773 −2 | 100 ± 3 | 100 ± 3 | 100 ± 2 | 100 ± 2 | 100 ± 1 | 100 ± 1 | 97 ± 1 | 97 ± 2 | 97 ± 2 | 96 ± 2 | 100 |
| 30 | 2.8657 −2 | 95 ± 7 | 96 ± 6 | 98 ± 5 | 100 ± 2 | 101 ± 1 | 101 ± 1 | 100 ± 1 | 100 ± 2 | 100 ± 3 | 100 ± 5 | 100 |
| 40 | 1.8895 −2 | 89 ± 7 | 91 ± 9 | 96 ± 9 | 105 ± 6 | 108 ± 1 | 108 ± 1 | 107 ± 1 | 106 ± 5 | 97 ± 8 | 95 ± 6 | 100 |
| 50 | 1.1709 −2 | 89 ± 6 | 91 ± 6 | 97 ± 7 | 108 ± 6 | 116 ± 2 | 116 ± 2 | 111 ± 3 | 108 ± 6 | 100 ± 5 | 98 ± 4 | 100 |
| 60 | 7.2589 −3 | 88 ± 6 | 90 ± 5 | 97 ± 5 | 105 ± 5 | 113 ± 5 | 113 ± 4 | 110 ± 3 | 107 ± 3 | 102 ± 3 | 100 ± 3 | 100 |
| 70 | 4.4787 −3 | 88 ± 4 | 93 ± 4 | 98 ± 4 | 101 ± 3 | 104 ± 3 | 104 ± 3 | 106 ± 2 | 106 ± 2 | 104 ± 2 | 103 ± 2 | 101 |
| 80 | 2.7576 −3 | 86 ± 4 | 93 ± 4 | 98 ± 4 | 98 ± 3 | 101 ± 3 | 101 ± 2 | 104 ± 2 | 106 ± 2 | 106 ± 2 | 107 ± 2 | 99 |
| 90 | 1.7100 −3 | 84 ± 5 | 92 ± 5 | 97 ± 4 | 96 ± 4 | 100 ± 3 | 100 ± 2 | 103 ± 2 | 107 ± 2 | 108 ± 2 | 110 ± 2 | 99 |
| 100 | 1.0676 −3 | ... | 89 ± 6 | 96 ± 6 | 96 ± 4 | 99 ± 4 | 99 ± 3 | 104 ± 3 | 107 ± 2 | 108 ± 2 | ... | 97 |
| 110 | 6.6470 −4 | ... | 87 ± 6 | 95 ± 6 | 96 ± 5 | 101 ± 5 | 101 ± 4 | 104 ± 3 | 109 ± 3 | 108 ± 3 | ... | 97 |
| 120 | 4.1506 −4 | ... | 85 ± 7 | 93 ± 7 | 97 ± 6 | 103 ± 6 | 103 ± 4 | 106 ± 3 | 111 ± 3 | 110 ± 3 | ... | 98 |
| 130 | 2.6353 −4 | ... | 84 ± 8 | 92 ± 7 | 100 ± 7 | 105 ± 6 | 105 ± 4 | 109 ± 4 | 114 ± 4 | 114 ± 3 | ... | 100 |
| 140 | 1.6994 −4 | ... | 82 ± 8 | 89 ± 8 | 100 ± 7 | 106 ± 6 | 106 ± 5 | 111 ± 4 | 114 ± 4 | 116 ± 4 | ... | 102 |
| 150 | 1.1119 −4 | ... | 80 ± 9 | 89 ± 9 | 100 ± 8 | 107 ± 7 | 107 ± 6 | 111 ± 5 | 116 ± 4 | 118 ± 4 | ... | 103 |
| 160 | 7.4713 −5 | ... | 76 ± 10 | 87 ± 9 | 100 ± 9 | 108 ± 8 | 108 ± 6 | 111 ± 6 | 117 ± 5 | 120 ± 5 | ... | 105 |
| 170 | 5.1159 −5 | ... | 74 ± 10 | 87 ± 10 | 100 ± 9 | 108 ± 8 | 108 ± 7 | 112 ± 7 | 119 ± 6 | 120 ± 6 | ... | 107 |
| 180 | 3.5578 −5 | ... | 72 ± 10 | 87 ± 10 | 100 ± 10 | 109 ± 8 | 109 ± 8 | 112 ± 7 | 120 ± 7 | 122 ± 7 | ... | 109 |
| 190 | 2.4663 −5 | ... | 70 ± 10 | 87 ± 10 | 100 ± 10 | 109 ± 8 | 109 ± 8 | 112 ± 8 | 123 ± 8 | 122 ± 8 | ... | 113 |
| 200 | 1.6957 −5 | ... | 69 ± 10 | 86 ± 10 | 100 ± 10 | 108 ± 8 | 108 ± 8 | 111 ± 8 | 120 ± 8 | 124 ± 8 | ... | 116 |

[a] Power of ten by which preceding figure should be multiplied.

Missile Command "ringbook" series[5] giving cumulative percentages of densities at 1 km intervals over eight stations (years 1951–1957), and 3) Naval Weapons Laboratory reports[6] giving standard deviations of density about seasonal mean values at 9 levels over 25 Russian stations, 40 Eurasian stations, and 6 Atlantic Missile Range stations (various periods, 1950–1962).

Standard deviations of density at each level were divided by the corresponding USSA density and then multiplied by 200 to approximate the 95% range density variabilities (±) about the monthly mean (i.e., the range that is not exceeded more than 5% of the time in the indicated month). Similarly, the difference between the 2.5 and 97.5% cumulative frequencies of densities (from the ringbooks) was likewise adjusted to USSA density to obtain a generally equivalent expression for the 95% range. For example, at 100 kft in January at 60 N, the density is 89 ± 6% of USSA, so that the 95% range is from about 83 to 95% of the USSA density.

Data in the Naval Weapons Laboratory (NWL) series are tabulated by seasons, rather than for individual months; hence they were not used directly. Instead, they were compared to Table 1 variabilities after it had been compiled; results were gratifying in most cases. Reconciliation of the reported values required a considerable amount of meteorological judgment and reference to other variability studies, such as those of Smith and Chenoweth[7] and Smith.[8]

All of these tabulations were based on radiosonde data for the lower 100 kft of the atmosphere. For higher regions, heavy reliance was put on standard deviations of density obtained from rocket observations over Eglin Air Force Base in 1961 and 1962, as summarized by Cole and Kantor.[9] Also used were data tabulated by Thiele[10] for White Sands and by Quiroz, Lambert, and Dutton[11] for all available rocketsonde stations.

For heights ≤ 100 kft, the reliability of the variability figures in Table 1 is considered good; the actual variability ranges represented as 95% presumably are between 92 and 98%. Above 100 kft, however, they are only fair estimates in summer and educated guesses in winter. At 200 kft in winter, the 95% range may be as little as ±5% or as much as ±30%.

Variability is primarily due to alternations of air masses as weather patterns change. Diurnal fluctuations may also be important, but Nee[12] estimates diurnal density variability over New England in spring as less than 1% of the daily mean.

## Correlations

Variations in density profiles due to compensating mechanisms in the atmosphere lead to alternate layers of above-normal and below-normal density.[8, 10, 13, 14] Consequently, for no location can the (+) values of variability from Table 1 be added to the base values at all altitudes to give a single, representative "high-density" atmosphere, nor can the (−) values be used to give a representative "low-density" atmosphere. Alternations of above-normal and below-normal density layers can be described, to some extent, by a correlation function that seems to decrease exponentially to zero over thicknesses of 25 to 40 kft, beyond which it becomes negative; i.e., above-normal density at one level is accompanied by below-normal density at levels 25 to 50 kft below it or above it. When the correlation $r_{xy}$ between the densities $x$ and $y$ at two different levels is known, the departure of $y$ from its mean can be estimated from the departure of $x$

$$(y - \bar{y})/s_y = r_{xy}(x - \bar{x})/s_x$$

where $\bar{x}$ and $\bar{y}$ are the means and $s_x$ and $s_y$ the standard deviation of $x$ and $y$. Unfortunately, correlations are generally available only up to about 70 kft[4, 6] and not in sufficient detail to provide a useful estimate of exactly how the correlation $r_{xy}$ varies with height or with the separation of $x$ and $y$. At still higher levels, correlation at 160 to 220 kft over Eglin Air Force Base have been derived from 18 soundings in spring and autumn.[9] These indicate a somewhat smaller decrease of correlation with thickness than at lower levels, e.g., $r = 0.7$ through a thickness of 30 kft. These data are too limited for any direct application in the construction of extreme density profiles, but they do confirm the general picture of alternating compensations.

## Conclusion

Atmospheric density decreases with height almost logarithmically, so that graphic representation of the actual density profile is difficult[15] and is more effective in terms of departure from some average or standard. Most tabulations and representations[4, 6, 8] have been in terms of averages for the location being studied. The diagram presented here places all of these local averages and variabilities on a common base. Although actual local values cannot be recovered from Fig. 1 and Table 1 with the same precision as from other approaches, the general pattern of the distribution of density and its variations emerges more clearly, and the resulting picture should be useful in vehicle designing.

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# Measurement of Aerodynamic Heating of Wind-Tunnel Models by Means of Temperature-Sensitive Paint

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### Nomenclature

$h$ = heat-transfer coefficient, Btu/ft$^2$-sec-°R
$M$ = Mach number
$Re$ = Reynolds number/ft, ft$^{-1}$
$t$ = time, sec
$\alpha$ = angle of attack, deg
$\beta$ = angle of yaw, deg
$\theta$ = complement of flow deflection angle, deg

***Subscripts***

$\infty$ = freestream conditions
$s$ = stagnation point
$0$ = stagnation conditions behind a normal shock in test section on 0.375-in. radius sphere

### Introduction

TEMPERATURE-SENSITIVE paints have been used for several years to obtain a visualization of the aerodynamic heating distribution on wind-tunnel models. An extensive series on X-20 glider models was done in 1961 in the Arnold Center Von Karman Facility C Tunnel at $M = 10.2$ and $Re = 2.2 \times 10^6$/ft. A four-color-change paint was used. Efforts were undertaken subsequently at The Boeing Company to obtain not only qualitative but also quantitative information from thermal paint tests. A method developed by Lorenz and Sartell[1] was successfully applied to some simple configurations. It is the purpose of this note to illustrate the application of the method to a complicated configuration where the results can be checked by thermocouple measurements.

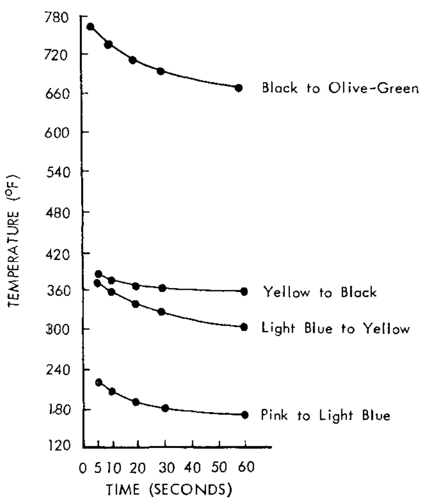


**Fig. 1 Exposure required to change color of Detectotemp 915-0933 paint.**

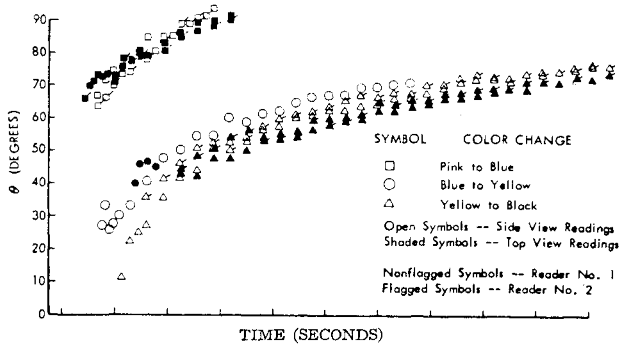


**Fig. 2 Six-inch sphere color change history.**

### Method and Models

The testing procedure is as follows:

1) The model and one or several calibration spheres are painted. A four-color-change paint was selected for this test. The paint calibration diagram (color-change temperature vs exposure time) is shown in Fig. 1.

2) The model and calibration spheres are separately injected into the wind tunnel, and color movies are taken of the color changes.

3) The calibration-sphere film is scanned frame by frame to establish the position $\theta$ of the boundaries between colors (e.g., pink to blue) vs time (see upper part of Fig. 2).

4) The $\theta$ vs $t$ curves are cross plotted against the theoretical $h/h_s$ vs $\theta$ curve for the sphere.[2] This leads to three curves of $h/h_0$ vs $t$, or frame number (lower part of Fig. 2). The fourth color change, black to olive, could not be observed. These are the final calibration curves that will be used for measuring $h/h_0$. — $h_0$ is a reference quantity defined as the heat-transfer coefficient at the stagnation point of a 0.75-in.-diam sphere under the same flow conditions ($h_0 = 0.032$ Btu/ft$^2$-sec-°R).

5) The model film is then scanned until that particular frame is found where the first color boundary (e.g., pink to blue) passes through a chosen station. The corresponding value of $h/h_0$ is then read from Fig. 3.

This procedure is repeated as successive color boundaries pass through the station, so that, in principle, four values of $h/h_0$ should be read for each station. These values should coincide or be very close. If significant discrepancies exist, they are due to conduction differences between the model and the sphere. In this case the first color change that is

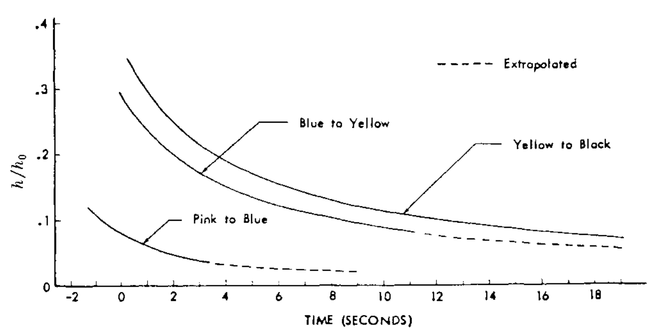


**Fig. 3 Heat-transfer calibration curves.**

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